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FELINES - Forecast of the Effects of Lightning IN Electrical Systems

# PBP Modeling and Analysis

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## 1 Introduction

The research project Forecast of the Effects of Lightning IN Electrical Systems (FELINES) aims at designing a protection system capable of sensing electromagnetic fields that are preliminary to a lightning event, and consequently disconnect part (or all) of the electric infrastructure under its protection. These fields are generated by the so-called Preliminary Breakdown Pulses (PBP), localized events taking place during the first phases of the lightning inception.

In this framework, we are dealing with the work package #2, which is devoted to develop a model for PBPs. We performed a literature review to find out a possible modelling approach. The model to implement must be able to obtain EM fields, induced voltages and currents describing the phenomenon with a level of detail and reliability suitable for the aims of the project. Such a model will be integrated with that describing the return stroke, looking for correlation between the PBP electromagnetic characteristics and the main parameters of the relevant return stroke. For this reason, the output of the models will be compared with field measurements.

Among the possible PBP modeling approaches that we have investigated, that proposed in [1] (labelled from now on as "MTLK model") has been chosen. The main advantage of the MTLK model [1] is the analogy with typical transmission line engineering models for the return stroke current (e.g., the Modified Transmission Line model with Exponential decay, MTLE [2]). The goal of our research is to determine the optimum parameters of the MTLK model current $i(z,t)$ [1] by fitting experimental data (i.e., minimizing the error between the computed E-fields and the measured PBP signals [3]), in order to develop a reliable and relatively fast simulation tool. In particular, the statistical distributions of model parameters will be the output of this work stage. This in principle would allow to generate a database of random PBP EM fields, which will be useful for the implementation of the next phases of the project.

The optimization procedure will be obtained with the MACACO toolbox (based on the Ant Colony Optimization) which allows, for each event, to find the set of parameters which best fit the real data.

## 2 Implementation of the MTLK model for Preliminary Breakdown Pulses of CG Lightning Flashes

The MTLK model for PBPs considers the PBP discharge channel as a vertical thin-wire radiator (transmission line) extending between the altitudes $H_1$ and $H_2$ and assumes the traveling current as a continuous function of time $t$ and height $z$. Such a current is expressed as

$$i(z,t) = f(z) I\left(t - \frac{z - H_1}{v}\right) \quad t \geq 0 \quad H_1 < z < H_2 \tag{1}$$

where $I$ is the current injected at the bottom of the discharge channel located at $H_1$ and traveling upward with a constant speed $v$ (lower than the speed of light in vacuum $c$), and $f$ models the variations of the current depending on on $z$. For example, for the MTLE model $f(z) = e^{\frac{-(z-H_1)}{\lambda}}$, in which $\lambda$ is the (empirical) current decay constant. The MTLK model assumes the current pulse starting at zero when $z = H_1$, increasing until a certain height $H_m$ is reached, and decaying back to zero when $z = H_2$. Thus, assuming the Kumaraswamy distribution [4, 5] for the shape of the current traveling along the transmission line, the authors of [1] proposed

$$f(z) = \left(\frac{z - H_1}{H_2 - H_1}\right)^{a-1} \left[1 - \left(\frac{z - H_1}{H_2 - H_1}\right)^{a}\right]^{b-1} \quad H_1 < z < H_2 \tag{2}$$

where $a$ and $b$ are shape parameters.

As far as the current source $I$ at the bottom of the transmission line, in accordance with [6], an asymmetric Gaussian function is assumed in [1]. This function is expressed as

$$I(t) = \begin{cases} A e^{-[\alpha(t-t_1)]^2} & 0 \leq t \leq t_1 \\ A e^{-\left[\alpha \frac{(t-t_1)t_1}{(t_2-t_1)}\right]^2} & t > t_1 \end{cases} \tag{3}$$

where $t_2$ is the considered time window, $t_1$ is the rise time of the current pulse, and $A$ is the amplitude parameter. The parameter $\alpha$ controls the asymmetric shape of the current pulse; for computation purposes $\alpha_{ind} = \alpha t_2$ is used in place of $\alpha$. By applying the Position By Fast Antenna (PBFA) technique [7], it is possible to obtain the location of the maximum peak PBP current. Thus, for the MTLK model the PBFA $z$-coordinate prediction is $H_m$. Then, an optimization procedure is implemented to determine the values of the free parameters which are $H_1$, $H_2$, $a$, $b$, $t_1$, $t_2$, $\alpha$, $A$, and $v$. The cost function to minimize in the optimization problem is

$$\rho_{norm} = \frac{1}{N} \sum_{i=1}^{N} \sum_{n=1}^{L_i} \sqrt{\frac{1}{L_n} \left[\frac{E_{mi}(t_n) - E_{ci}(t_n)}{\Delta E_{mi,\,pp}}\right]^2} \tag{4}$$

where $N$ is the total number of E-field sensors and $L_i$ is the number of samples in waveforms recorded by the $i$-sensor. $E_{mi}(t_n)$ is the electric field measured by the $i$-sensor at $t = t_n$, and $E_{ci}(t_n)$ is the corresponding value given by computations. The normalizing factor $\Delta E_{mi,\,pp}$ is the peak-to-peak electric field measured at the $i$-sensor. The vertical electric filed produced by a current pulse traveling along a vertical radiator (transmission line) observed at $(x_0, y_0, z_0)$, is given by the Maxwell's equations formulation provided in [8]

$$E(D,z_0,t) = \frac{1}{2\pi\varepsilon_0}\int_{H_1}^{H_2}\frac{2(z_0-z)^2-D^2}{R^5}\int_0^t i\left(z,\tau-\frac{R}{c}\right)d\tau dz+ \\ +\frac{1}{2\pi\varepsilon_0}\int_{H_1}^{H_2}\frac{2(z_0-z)-r^2}{cR^4}\,i\left(z,t-\frac{R}{c}\right)dz+ \\ -\frac{1}{2\pi\varepsilon_0}\int_{H_1}^{H_2}\frac{r(z_0-z)}{c^2R^3}\,\frac{\partial i\,(z,t-R/c)}{\partial t}dz \quad (5)$$

where $\varepsilon_0$ is electric permittivity of vacuum, $D$ is the horizontal distance between the vertical radiator and the observation point, and $R=\sqrt{(z_0-z)^2+D^2}$ is the distance between the field source and the observation point.

### 2.1 Reproduction of the MTLK model result

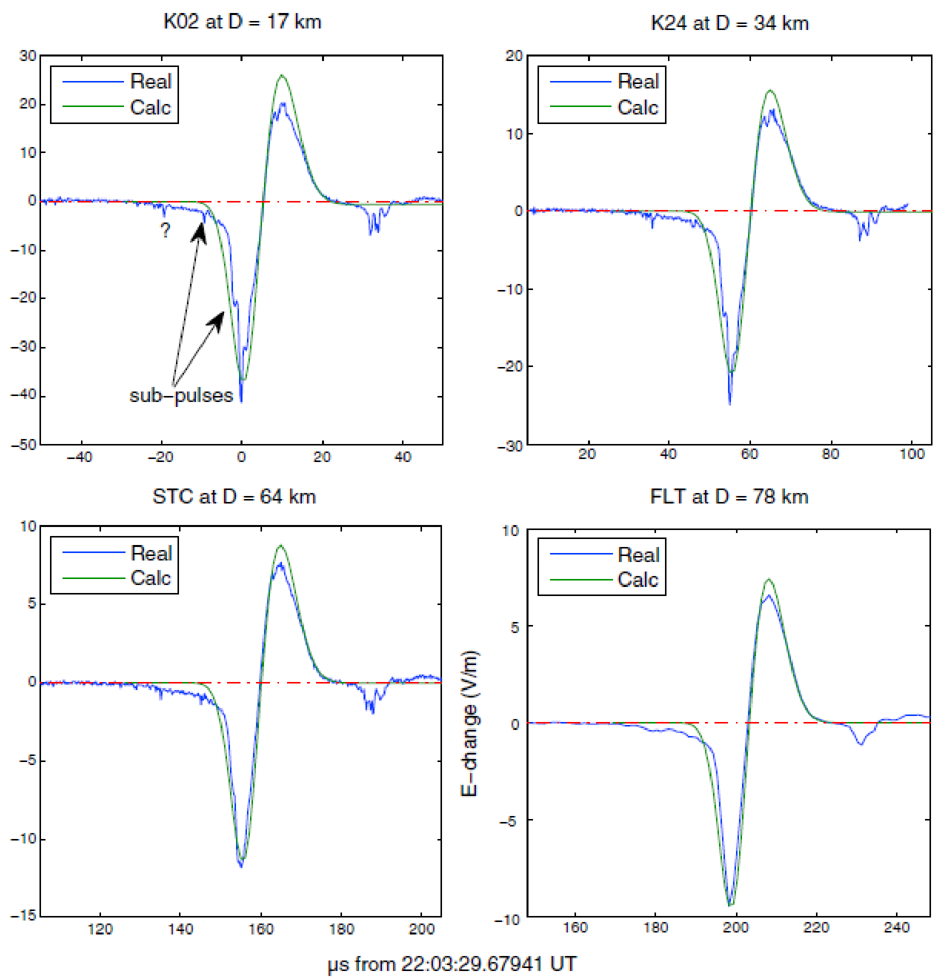



Figure 1: Results of [1]. Waveform comparison of MTLK results (shown in green) to measured data (blue) shown for the PBP-4 at four different locations.

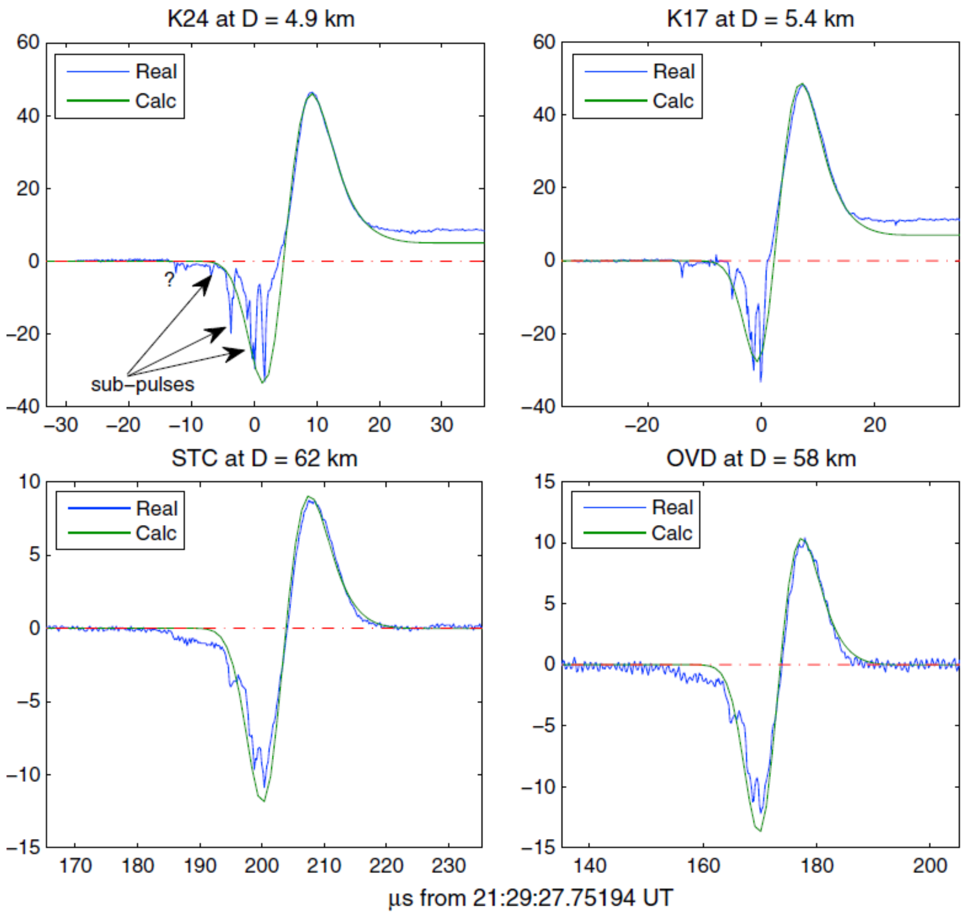



Figure 2: Results of [1]. Waveform comparison of MTLK results (shown in green) to measured data (blue) shown for the PBP-2 at four different locations.

The following procedure is implemented:

- Waveform samples $E_{mi}(t_n)$ from two different PBPs are extrapolated from Figures 4 and 6 of [1], which are reported here in Figures 1 and 2 (blue curves), respectively.
- Set of the maximum $f$ condition. Introducing the linear transformation $x = \frac{z-H_1}{H_2-H_1}$, equation (2) becomes

$$f(x) = x^{a-1}\left(1-x^a\right)^{b-1} \quad 0 < x < 1 \tag{6}$$

Thus, its derivative is

$$f'(x) = -\frac{x^{a-1}\left(1-x^a\right)^{b-1}\left(bax^a - x^a - a + 1\right)}{x\left(1-x^a\right)} = \\ = -x^{a-2}(1-x^a)^{b-2}\left[(ba-1)x^a + a - 1\right] \quad 0 < x < 1 \tag{7}$$

By imposing the condition $f'(x_m) = 0$ and considering

$$0 < x_m = \frac{H_m - H_1}{H_2 - H_1} < 1 \tag{8}$$

it is possible to obtain a relationship between the shape parameters $a$ and $b$, i.e.

$$b = \frac{x_m^a + a - 1}{a x_m^a} \tag{9}$$

- input data in terms of sensor distance from the PBP event are provided (Table 1)
- Optimization tool: Multivariate Ant Colony Algorithm for Continuous Optimization (MACACO) [9]. This allows to find the best set of parameters which fit the data.

Results are reported in Figures 3 and 4, for the PBP-4 and the PBP-2, respectively. The corresponding model parameters are reported in Table 1, where also the parameters by the authors of [1] are proposed. It is worth to note that, to keep a reduced CPU time, a large time step has been considered ($10^{-6}$ s). In these conditions, the cost function obtained with MACACO is much higher than that of [1]. Moreover, the optimum parameters are completely different between the two methods, suggesting that the same field shape can be reproduced with different combinations of the parameters. This is an important issue which should be addressed in details and at a statistical level with the tests on a large number of cases.

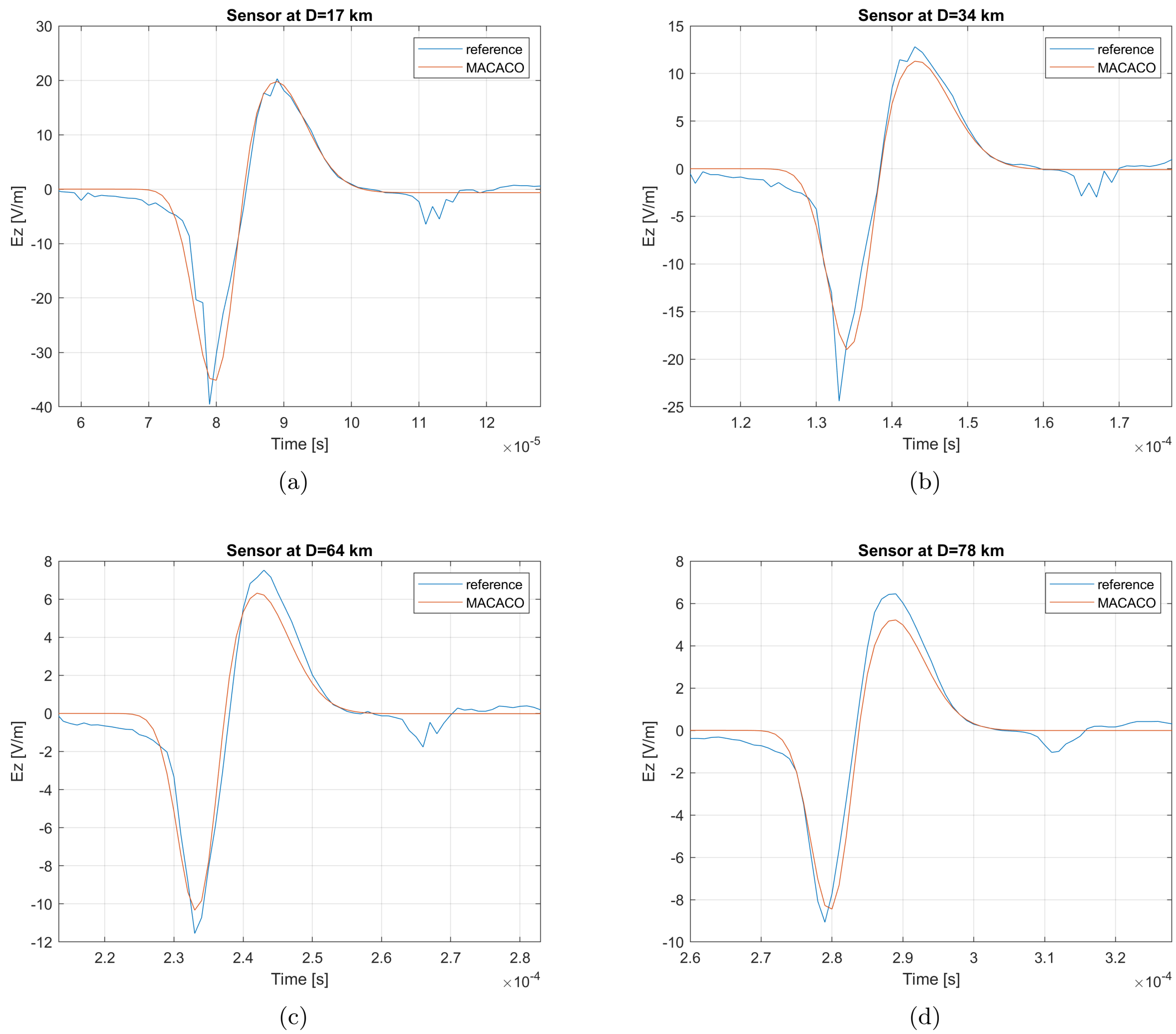



Figure 3: Results with MACACO (time step $10^{-6}$ s). Waveform comparison of MTLK results (shown in red) to measured data (blue) shown for the PBP-4 at four different sensor locations.

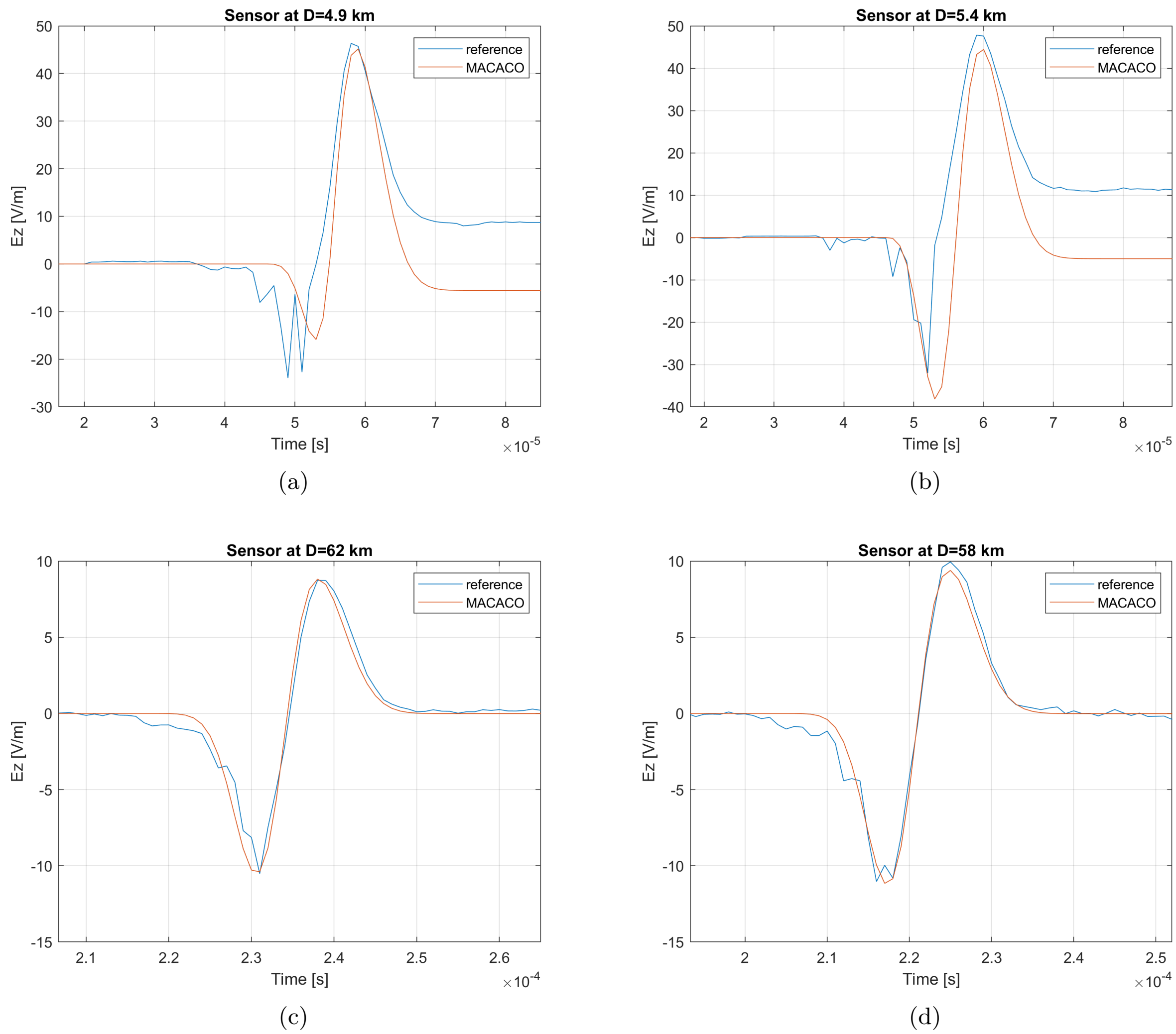



Figure 4: Results with MACACO (time step $10^{-6}$ s). Waveform comparison of MTLK results (shown in red) to measured data (blue) shown for the PBP-2 at four different sensor locations.

Table 1: Input (in parenthesis) and output parameters for PBP-4 and PBP-2 MTLK models as reported in [1] and as caluclated with MACACO.

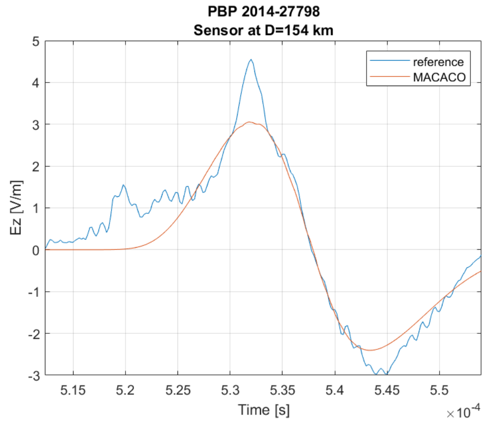
| Parameter | PBP-4 Reference [1] | PBP-4 MACACO | PBP-2 Reference [1] | PBP-2 MACACO |
|---|---|---|---|---|
| $x_0$ (m) | (-24653) | (-24653) | (-952) | (-952) |
| $y_0$ (m) | (-20106) | (-20106) | (-1902) | (-1902) |
| $z_0$ (m) | (0) | (0) | (0) | (0) |
| $H_m$ (m) | 5530 | 6120 | 5824 | 6221 |
| $H_1$ (m) | 5244 | 5974 | 5706 | 6017 |
| $H_2$ (m) | 6260 | 7266 | 6625 | 7360 |
| $t_1$ ($\mu$s) | 19.9 | 16.3 | 25 | 20 |
| $t_2$ ($\mu$s) | 46.7 | 52.04 | 46.7 | 49.2 |
| $v$ ($\times 10^8$ m/s) | 1.1 | 1.4 | 1.2 | 1.3 |
| $\alpha_{ind}$ | 9.2 | 13.67 | 8.7 | 13.22 |
| $a$ | 1.5 | 2.69 | 1.2 | 2.93 |
| $b$ | 2.9 | 3.38 | 2.9 | 3.35 |
| $A$ (kA) | 119 | 150 | 101 | 163 |
| $\rho_{norm}$ | $4.70 \times 10^{-2}$ | 0.14 | $4.60 \times 10^{-2}$ | 0.43 |

### 2.2 Implementation of the MTLK model with samples of the database of [3]

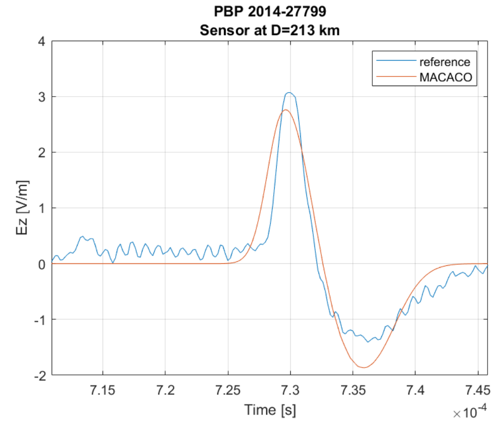
The same approach described in the previous subsection is applied to the five available PBP samples of the database presented and analyzed in [3]. Results are reported in Figures 5. The corresponding model parameters and their statistical distributions are reported in Table 2-3. Although the availability of only one sensor for each PBP measurement, results reveal good performance of the optimization tool, suggesting the possibility of implementing the procedure for a large database.

From the results of Figure 5 some cases are not well described by the proposed model (for example PBP 2014-28110). At this stage we do not have an answer for this behaviour but in the future steps we are going to propose a criterion to remove the events that lead to a poor reconstruction by means of the MTLK model in order to avoid errors in the statistical distributions of the MTLK parameters.

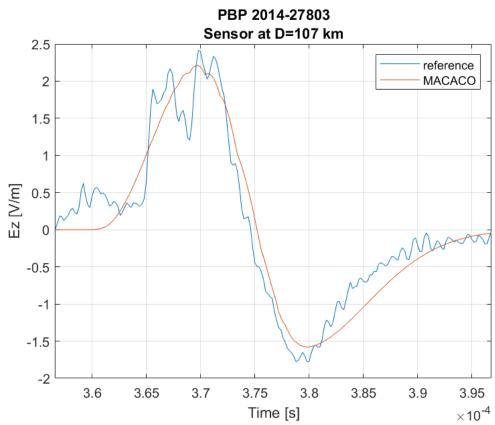
The statistical distribution of each parameter is presented in Table 3 in terms of mean and standard deviation. Clearly, these values are computed taking into account only 5 events, but they will be improved and refined when the whole database will be analyzed.



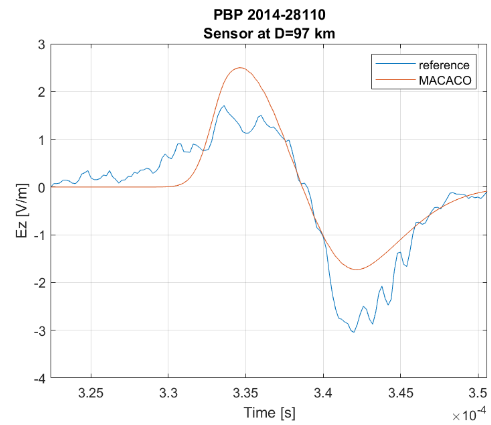

(a)



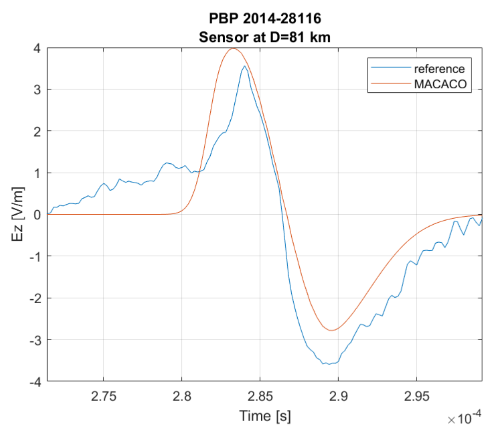

(b)




(c)




(d)




(e)

Figure 5: Results with MACACO (time step $10^{-6}$ s) considering 5 different events. Waveform comparison of MTLK results (shown in red) to measured data (blue) [3] shown for five PBP samples.

Table 2: Input (in parenthesis) and output parameters of the MTLK model for the five samples of the database of [3] as calculated with the optimization tool MACACO.

| Parameter | 2014-27798 | 2014-27799 | 2014-27803 | 2014-28110 | 2014-28116 |
|---|---|---|---|---|---|
| $D$ (m) | (153710) | (213270) | (106990) | (96730) | (81410) |
| $H_m$ (m) | 5563.1 | 5763.5 | 5980.6 | 5836.4 | 6018.5 |
| $H_1$ (m) | 5247.0 | 5172.9 | 5109.2 | 5265.2 | 5309.1 |
| $H_2$ (m) | 6247.1 | 6173.7 | 6346.5 | 6306.3 | 6230.2 |
| $t_1$ ($\mu$s) | 20.0 | 18.0 | 11.0 | 10.8 | 10.4 |
| $t_2$ ($\mu$s) | 46.2 | 46.4 | 45.9 | 46.1 | 46.0 |
| $v$ ($\times 10^8$ m/s) | 1.0 | 1.0 | 1.1 | 1.1 | 1.1 |
| $\alpha_{ind}$ | 6.6 | 26.9 | 17.1 | 28.5 | 34.2 |
| $a$ | 1.4 | 1.4 | 1.4 | 1.4 | 1.5 |
| $b$ | 2.9 | 3.0 | 2.9 | 2.9 | 2.9 |
| $A$ (kA) | -102.2 | -71.54 | -30.66 | -30.66 | -40.88 |
| $\rho_{norm}$ | $5.2\times 10^{-4}$ | $4.6\times 10^{-4}$ | $5.0\times 10^{-4}$ | $6.5\times 10^{-4}$ | $5.9\times 10^{-4}$ |

Table 3: Statistical distributions of the parameters of the MTLK model computed from the results of Table 2.

| Parameter | Mean | Standard Deviation |
|---|---|---|
| $H_m$ (m) | 5832 | 182.78 |
| $H_1$ (m) | 5220.5 | 79.51 |
| $H_2$ (m) | 6260.4 | 67.35 |
| $t_1$ ($\mu$s) | 14.04 | 4.58 |
| $t_2$ ($\mu$s) | 46.12 | 0.19 |
| $v$ ($\times 10^8$ m/s) | 1.06 | 0.05 |
| $\alpha_{ind}$ | 22.66 | 10.88 |
| $a$ | 1.42 | 0.04 |
| $b$ | 2.92 | 0.04 |
| $A$ (kA) | -55.19 | 31.17 |

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